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TURBULENT HEAT TRANSFER IN A CIRCULAR TUBE WITH VARIABLE CIRCUMFERENTIAL HEAT FLUX

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Abstract—An analysis for hydrodynamically and thermally fully developed heat transfer in a circular tube with variable circumferential heat flux is presented. The results allow prediction of temperature variations when the tube is heated uniformly in the axial direction and non-uniformly around its perimeter. A surprising conclusion is that the effects of circumferential heat flux variation in turbulent flow are sometimes more pronounced than in laminar flow. An example shows the striking importance of these effects.

NOMENCLATURE

A^+,	profile constant;
a_n, b_n,	Fourier coefficients;
C_p,	specific heat at constant pressure;
E,	$(1 + \epsilon_H/a)$;
f,	Fanning friction factor $\sqrt{(\tau_w)}/\frac{1}{2}\rho u_m^2$;
$F(\theta)$,	heat flux variation about the mean;
$g(r, \theta)$,	temperature difference function;
h,	convective heat transfer conductance, $q''/\Delta t_w$;
k,	thermal conductivity;
n,	harmonic index;
Nu_o,	mean Nusselt number, $2hr_o/k$;
Pr,	Prandtl number;
q'',	heat flux;
r,	radius co-ordinate;
r_o^+,	$r_o\sqrt{(\tau_w/\rho)}/\nu$;
r_o,	tube radius;
r^*,	r/r_o;
R_n,	radial eigenfunction;
Re,	Reynolds number, $2U_m r_o/\nu$;
S_n,	wall temperature functions;
t,	temperature;
u,	velocity;
y,	distance from wall;
y^+,	$y\sqrt{(t_w/\rho)}/\nu$.

Greek symbols

Δt,	temperature difference above mean;
a,	thermal diffusivity, $k/\rho C_p$;
ϵ_H,	eddy diffusivity for heat;
ϵ_M,	eddy diffusivity for momentum;
κ,	profile constant;
μ,	viscosity;
ν,	kinemetic viscosity, μ/ρ;
ρ,	density;
τ_w,	wall shear stress;
θ,	angular co-ordinate.

Subscripts

o,	average;
m,	mixed mean;
n,	harmonic;
w,	wall.

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INTRODUCTION

THE circular tube is a very common geometry employed in nuclear reactors, and often in this application the heat flux to the coolant varies considerably both along the tube and around its perimeter. These variations influence the Nusselt number, and consequently are of considerable practical importance. The effects of axial heat flux variation are usually rather small, except where very rapid changes occur [1]. However, the influence of circumferential variations is much more pronounced, as a previous laminar flow analysis indicated [2]. In this paper we report an analysis of the turbulent flow problem, in which the circumferential flux distribution is found to be surprisingly important. The laminar analysis is included here for completeness.

FORMULATION

We consider the case of hydrodynamically fully developed flow of a fluid having constant properties, and seek the thermally fully developed temperature field for flow in a circular tube with a prescribed wall heat flux. This flux can have any arbitrary circumferential distribution, but is invariant in the flow direction. The eddy diffusivity concept is useful here, and we shall base our analysis on the key assumption that the diffusivities for heat in the *radial* and *circumferential* directions are *identical*. This is somewhat like an isotropy idealization, and is unquestionably the assumption most subject to debate. However, no other idealization seems more appropriate in view of the unavailability of pertinent experimental data. Under these idealizations the differential equation governing the temperature field may be obtained in the usual way from a simple energy balance, and is

$$L(t) = u\frac{\partial t}{\partial x} \tag{1a}$$

where for brevity we have put

$$L(\ ) = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\alpha + \epsilon_H)\frac{\partial}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left[(\alpha + \epsilon_H)\frac{\partial}{\partial \theta}\right]. \tag{1b}$$

This equation is elliptic in r–θ, and the associated boundary condition is that the heat flux is prescribed,

$$q''_w(\theta) = \left(k\frac{\partial t}{\partial r}\right)_{r=r_o} \quad \text{(prescribed).} \tag{2a}$$

We may represent the prescribed heat flux in the form

$$q''_w(\theta) = q''_o + F(\theta) \tag{2b}$$

where

$$\int_0^{2\pi} F(\theta)\,\mathrm{d}\theta = 0\cdot \tag{2c}$$

Since our interest is with the developed temperature field, we will work only with the temperature difference above the mean fluid temperature, which can be determined from an overall energy balance. We denote

$$t(r, x, \theta) = t_m(x) + \Delta t(r, \theta). \tag{3}$$

Equation (1) then becomes

$$L(\Delta t) = u\frac{\mathrm{d}t_m}{\mathrm{d}x}. \tag{4}$$

Since the heat input is constant in the axial direction, $\mathrm{d}t_m/\mathrm{d}x$ is constant. We next split the temperature difference into two parts,

$$\Delta t(r, \theta) = \Delta t_o(r) + g(r, \theta). \tag{5}$$

The function $\Delta t_o(r)$ is taken to be the solution of (4) which satisfies the boundary condition

$$k\left(\frac{\mathrm{d}\Delta t_o}{\mathrm{d}r}\right)_{r=r_o} = q''_o \tag{6}$$

and is therefore the temperature field associated with the average heat flux q''_o. It is a particular solution, which takes care of the inhomogeneous term. The function $g(r, \theta)$ then does not contribute to bulk temperature rise, and satisfies the simpler elliptic equation

$$L(g) = 0. \tag{7}$$

The boundary condition on $g(r, \theta)$ becomes

$$k\left(\frac{\partial g}{\partial r}\right)_{r=r_o} = F(\theta). \tag{8}$$

It is indeed interesting that g is independent of the velocity field (but it does depend on the eddy diffusivity distribution).

We consider cases where the function $F(\theta)$ has a Fourier expansion, and put

$$F(\theta) = \sum_{n=1}^{\infty}(a_n \sin n\theta + b_n \cos n\theta). \tag{9}$$

The solution for $g(r, \theta)$ may then be obtained in the form

$$g(r, \theta) = \frac{r_o}{k}\sum_{n=1}^{\infty} R_n(r)\,(a_n \sin n\theta + b_n \cos n\theta). \tag{10}$$

We now introduce the dimensionless quantities

$$E(r^*) = (1 + \epsilon_H/\alpha) \tag{11a}$$

$$r^* = r/r_o \tag{11b}$$

and, using (7), find that the functions $R_n(r^*)$ must satisfy

$$\frac{d}{dr^*}\left(r^* E \frac{dR_n}{dr^*}\right) - n^2 \frac{E}{r^*} R_n = 0 \tag{12}$$

with the boundary conditions

$$R_n'(1) = 1 \tag{13a}$$

$$R_n(0) = 0, \text{ (for regular solutions).} \tag{13b}$$

We assume that the solutions are in hand, and denote

$$S_n = R_n(1). \tag{14}$$

In uniform heat flux analyses it is customary to define a Nusselt number, and in this case

$$Nu_o = (q_o''/\Delta t_{ow})\, 2r_o/k. \tag{15}$$

Assuming that the Nusselt number is known, the contribution of the function Δt_o to the local wall temperature difference may be written as

$$\Delta t_{ow} = S_o q_o'' r_o/k \tag{16a}$$

where

$$S_o = 2/Nu_o. \tag{16b}$$

The local wall temperature corresponding to the arbitrarily prescribed heat flux is therefore

$$t_w(\theta, x) - t_m(x)$$

$$= \frac{r_o}{k}\left[S_o q_o'' + \sum_{n=1}^{\infty} S_n(a_n \sin n\theta + b_n \cos n\theta)\right]. \tag{17}$$

We see that once the values of the S_n's have been determined we can calculate the temperature difference for any prescribed circumferential heat flux.

LAMINAR FLOW

The laminar solutions may be obtained by setting $E = 1$.

One obtains

$$R_n(r^*) = r^{*n}/n \tag{18a}$$

from which

$$S_n = 1/n, \quad n > 0. \tag{18b}$$

The Nusselt number for laminar flow in a circular tube with constant heat flux is well known [3], and is

$$Nu_o = 48/11. \tag{18c}$$

Therefore, for laminar flow,

$$S_o = 11/24 = 0{\cdot}458. \tag{18d}$$

Note that S_1 exceeds S_o by more than a factor of 2! This shows the important influence of circumferential heat flux variation on the convection process.

TURBULENT FLOW

Although the velocity does not appear in the differential equation of the R_n's (12), the eddy diffusivity for heat is involved, and an appropriate representation must be employed. The term E may be rewritten as

$$E = 1 + \frac{\epsilon_M}{\nu}\frac{\epsilon_H}{\epsilon_M} Pr. \tag{19}$$

In the present numerical solutions an expression for the eddy diffusivity for momentum suggested by Cess [4] was employed. This expression represents a combination of a sublayer equation due to van Driest [5] and a middle law suggested by Reichardt [6], and is

$$\frac{\epsilon_M}{\nu} = \frac{1}{2}\left\{1 + \frac{\kappa^2 (r_o^+)^2}{9}\left[1 - \left(\frac{r}{r_o}\right)^2\right]^2 \left[1 + 2\left(\frac{r}{r_o}\right)^2\right]^2 \left[1 - \exp\left(-\frac{1 - r/r_o}{A^+/r_o^+}\right)\right]^2\right\}^{\frac{1}{2}} - \frac{1}{2} \tag{20}$$

with

$$A^+ = 26, \tag{21a}$$

$$\kappa = 0 \cdot 4, \tag{21b}$$

$$r_o^+ = r_o\sqrt{(\tau_w/\rho)}/\nu = Re\sqrt{(f/8)}. \tag{21c}$$

The friction factor appearing in (21c) was evaluated from the power-form expressions listed below:†

$$f = 0 \cdot 079\ Re^{-0 \cdot 25},\quad 5000 \leqslant Re \leqslant 30\,000, \tag{22a}$$

$$f = 0 \cdot 046\ Re^{-0 \cdot 2},\quad 30\,000 \leqslant Re \leqslant 1\,000\,000. \tag{22b}$$

Jenkins [7] calculated the diffusivity ratio ϵ_H/ϵ_M, based on a very simple eddy model. Jenkins' results indicate that the ratio is less than unity for Prandtl numbers around 0·7, but measurements seem to indicate a ratio somewhat in excess of unity. Sleicher and Tribus [8] analysed the circular tube using Jenkins' analysis adjusted by a multiplying factor to give agreement with Sleicher's own measurements at the single Prandtl number of 0·7. Kays and Leung [9] recently employed Jenkins' curves with a smaller bumping factor, and obtained results which are in better agreement with subsequent experiments in this Prandtl number range and are not bad at low Prandtl numbers. In the present calculations at Prandtl numbers of 0·7 and below the Jenkins' ratios were multiplied by 1·15 to estimate ϵ_H/ϵ_M (this is essentially the procedure employed by Kays and Leung). At high Prandtl numbers the sublayers become controlling, and it does not make much difference in heat transfer calculations what diffusivity is employed outside the sublayers. The Cess expression and a corresponding sublayer expression of Deissler [10] are quite similar, and constants in the Deissler expression were basically determined from high Prandtl number heat transfer data using a heat transfer analysis which assumed unity diffusivity ratio. The Deissler diffusivity should therefore be interpreted as thermal diffusivity for high Prandtl numbers. It was felt that a reasonable procedure for the high Prandtl number calculations would be to simply use the Cess expression, and a diffusivity ratio of 1·15 and this was the method employed for $Pr > 3$.

† In view of the other simplifying idealizations, these expressions were deemed adequate.

The radial temperature functions $R_n(r^*)$ for $n \geqslant 1$ were obtained numerically on a Burroughs 220 digital computer. Equation (12) was reduced to a pair of simultaneous first order differential equations, and these were solved using a fourth-order Adams predictor-corrector method [11]. The calculation was handled as an initial value problem from the center. Since the diffusivity function E is quite flat at the center, the turbulent solutions behave like the laminar solutions, i.e. $R_n(r^*)$ behaves like r^{*n}. This fact was used in starting the integration procedure, which was carried to the wall. The homogeniety of equation (12) permits multiplication of any solution by a constant, and the functions computed from the integration could therefore be normalized to make $R_n'(1) = 1$. The integrations were performed using eighty increments, distributed as shown below:

$$0 \leqslant y^+ \leqslant 10, \quad 20 \text{ increments}$$
$$10 \leqslant y^+ \leqslant 60, \quad 20 \text{ increments}$$
$$60 \leqslant y^+ \leqslant 160, \quad 20 \text{ increments}$$
$$160 \leqslant y^+ \leqslant r_o^+, \quad 20 \text{ increments.}$$

Convergence was checked by calculations using twice as many increments. The most error occurs at high Reynolds and Prandtl numbers, where the results reported are converged within 1 per cent. Below $Pr = 100$ the calculation is accurate to better than 0·1 per cent. The calculation was also checked by computing the laminar functions, and six-figure agreement was obtained for the first five harmonics.

A typical diffusivity (E) distribution is shown in Fig. 1. Note that it varies only a little over most of the flow. Typical radial functions as normalized to give $R_n'(1) = 1$ are shown in Fig. 2. There is a marked similarity to the form of laminar functions (18). It is interesting to note that for $Pr = 0$ the turbulent radial functions become identical with the laminar functions, since $E = 1$ for either case.

The values of $S_o(Re, Pr)$ were computed from

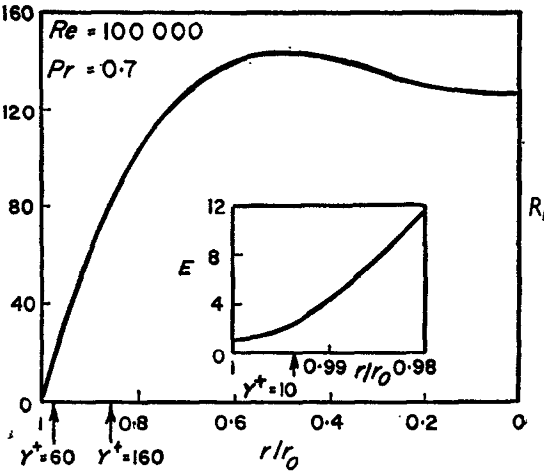

FIG. 1. A typical diffusivity distribution.

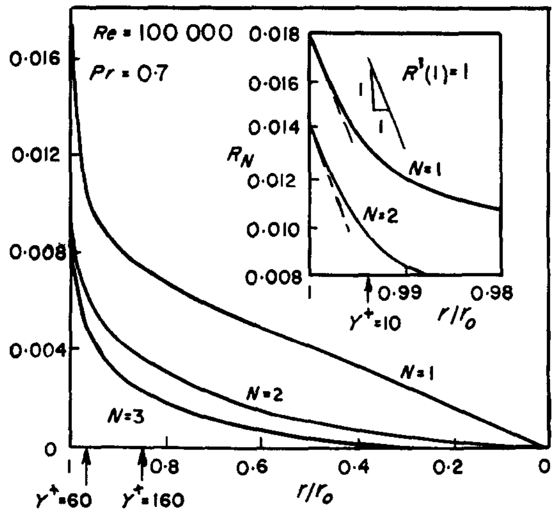

FIG. 2. Typical radial eigenfunctions.

the Nusselt numbers given by Kays and Leung (9). These calculations represent a self-consistent treatment of fully developed turbulent heat transfer in a circular tube with uniform axial flux, and the diffusivity assumptions which they employed are basically those employed in the present calculations. The values are shown in Fig. 3, and included in Table 1.

The values of $S_n(Re,Pr)$ were computed for the first five harmonics for a series of Reynolds and Prandtl number, and the results are summarized in Table 1. A partial presentation of these functions is shown in Fig. 4.

Table 1. *Circumferential heat flux functions* $S_n(Re, Pr)$

Pr	*n*	*Re*				
		10^4	3×10^4	10^5	3×10^5	10^6
0	0	0·318	0·302	0·293	0·288	0·283
	1	1·000	1·000	1·000	1·000	1·000
	2	0·500	0·500	0·500	0·500	0·500
	3	0·333	0·333	0·333	0·333	0·333
	4	0·250	0·250	0·250	0·250	0·250
	5	0·200	0·200	0·200	0·200	0·200
0·001	0	0·318	0·302	0·293	0·282	0·246
	1	1·000	1·000	0·999	0·974	0·901
	2	0·500	0·500	0·499	0·491	0·469
	3	0·333	0·333	0·333	0·329	0·320
	4	0·250	0·250	0·250	0·248	0·244
	5	0·200	0·200	0·200	0·199	0·196
0·003	0	0·318	0·302	0·282	0·246	0·156
	1	0·999	0·994	0·957	0·831	0·473
	2	0·500	0·498	0·484	0·435	0·279
	3	0·333	0·332	0·325	0·299	0·203
	4	0·250	0·249	0·245	0·229	0·170
	5	0·200	0·200	0·197	0·186	0·145

Table 1.—*continued*

Pr	*n*	*Re*				
		10^4	3×10^4	10^5	3×10^5	10^6
0·01	0	0·311	0·286	0·224	0·141	0·0655
	1	0·991	0·952	0·733	0·409	0·161
	2	0·497	0·483	0·397	0·246	0·109
	3	0·332	0·325	0·279	0·186	0·0894
	4	0·249	0·245	0·217	0·153	0·0784
	5	0·199	0·197	0·178	0·132	0·0710
0·03	0	0·290	0·220	0·126	0·0618	0·0248
	1	0·923	0·699	0·348	0·145	0·0535
	2	0·473	0·383	0·214	0·0986	0·0402
	3	0·302	0·272	0·165	0·0816	0·0353
	4	0·243	0·213	0·138	0·0720	0·0326
	5	0·195	0·176	0·120	0·0654	0·0307
0·7	0	0·0631	0·0283	0·0112	0·00465	0·00174
	1	0·121	0·0490	0·0180	0·00721	0·00275
	2	0·0900	0·0378	0·0141	0·00578	0·00226
	3	0·0784	0·0336	0·0127	0·00525	0·00209
	4	0·0716	0·0313	0·0119	0·00496	0·00199
	5	0·0668	0·0297	0·0114	0·00477	0·00193
3	0	0·0325	0·0134	0·00495	0·00194	0·0$_3$690
	1	0·0448	0·0178	0·00629	0·00246	0·0$_3$902
	2	0·0379	0·0151	0·00540	0·00213	0·0$_3$791
	3	0·0353	0·0142	0·00508	0·00201	0·0$_3$751
	4	0·0338	0·0137	0·00490	0·00194	0·0$_3$728
	5	0·0327	0·0133	0·00479	0·00190	0·0$_3$714
10	0	0·0201	0·00806	0·00290	0·00111	0·0$_3$383
	1	0·0239	0·00931	0·00322	0·00123	0·0$_3$438
	2	0·0218	0·00853	0·00296	0·00113	0·0$_3$405
	3	0·0211	0·00824	0·00286	0·00110	0·0$_3$393
	4	0·0206	0·00808	0·00281	0·00108	0·0$_3$386
	5	0·0203	0·00797	0·00277	0·00107	0·0$_3$382
30	0	0·0142	0·00553	0·00194	0·0$_3$727	0·0$_3$248
	1	0·0151	0·00582	0·00199	0·0$_3$751	0·0$_3$261
	2	0·0144	0·00556	0·00190	0·0$_3$718	0·0$_3$250
	3	0·0141	0·00546	0·00187	0·0$_3$706	0·0$_3$246
	4	0·0140	0·00541	0·00185	0·0$_3$699	0·0$_3$244
	5	0·0139	0·00537	0·00184	0·0$_3$695	0·0$_3$242
100	0	0·00975	0·00384	0·00132	0·0$_3$496	0·0$_3$167
	1	0·00994	0·00383	0·00130	0·0$_3$486	0·0$_3$166
	2	0·00973	0·00375	0·00127	0·0$_3$476	0·0$_3$163
	3	0·00965	0·00372	0·00126	0·0$_3$472	0·0$_3$162
	4	0·00960	0·00370	0·00126	0·0$_3$470	0·0$_3$161
	5	0·00957	0·00369	0·00125	0·0$_3$469	0·0$_3$161
1000	0	0·00527	0·00205	0·0$_3$706	0·0$_3$265	0·0$_4$885
	1	0·00513	0·00198	0·0$_3$667	0·0$_3$248	0·0$_4$841
	2	0·00511	0·00197	0·0$_3$665	0·0$_3$247	0·0$_4$838
	3	0·00510	0·00196	0·0$_3$664	0·0$_3$247	0·0$_4$837
	4	0·00509	0·00196	0·0$_3$663	0·0$_3$247	0·0$_4$836
	5	0·00509	0·00196	0·0$_3$663	0·0$_3$247	0·0$_4$836

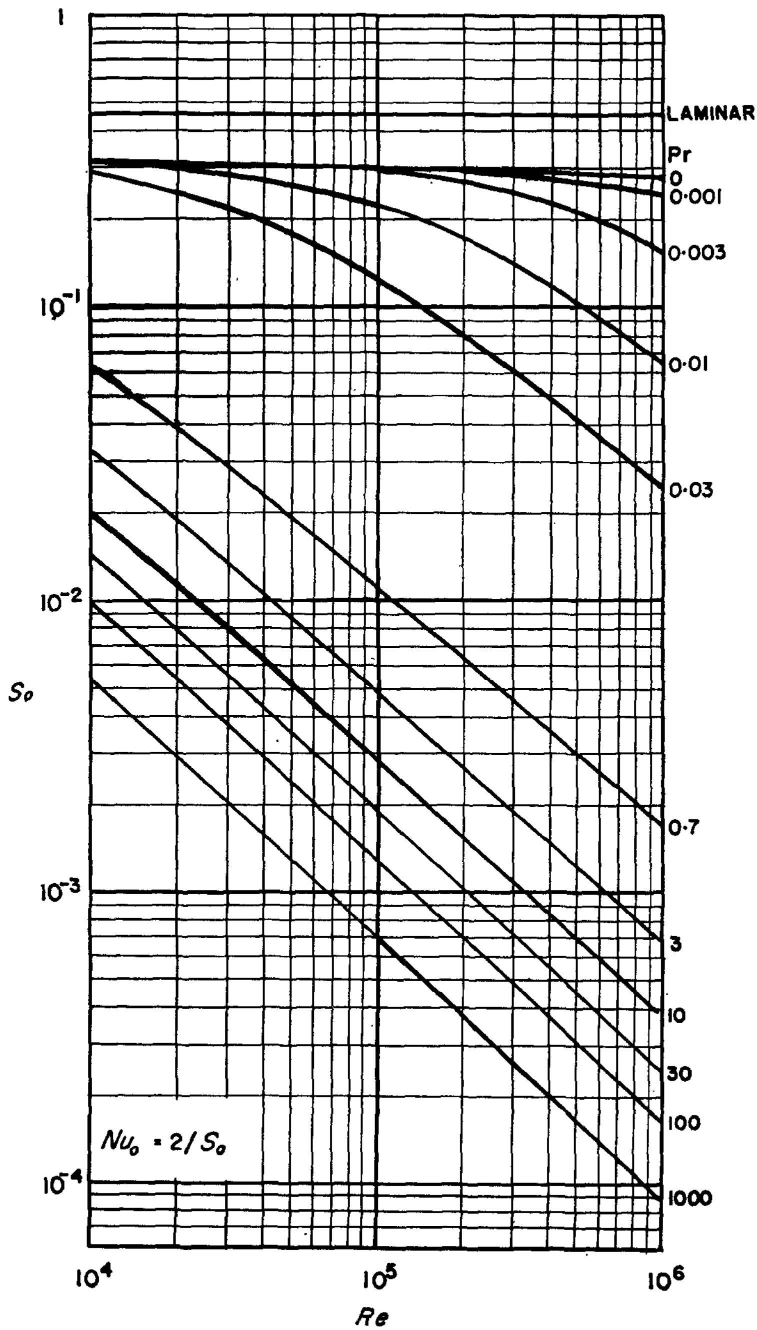

FIG. 3. Uniform heat flux functions.

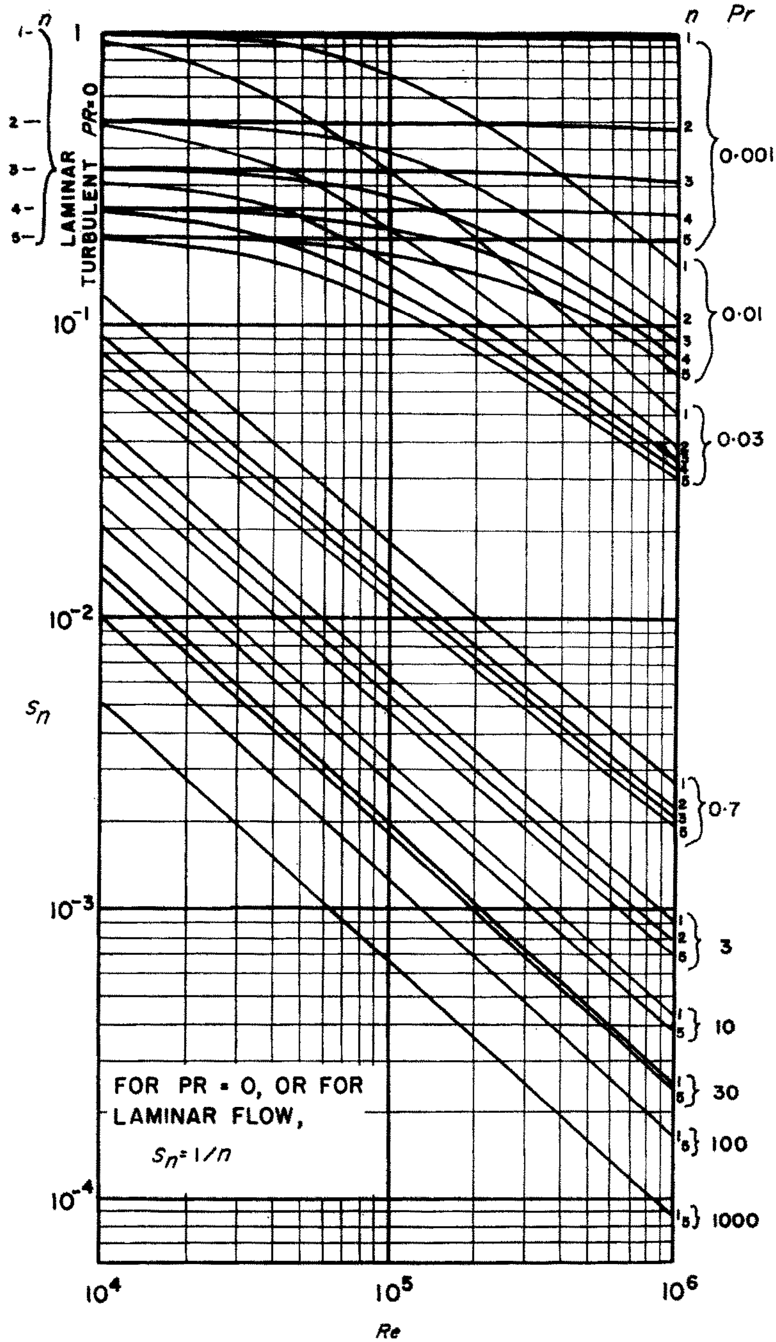

FIG. 4. Periodic heat flux functions.

DISCUSSION OF RESULTS

There are a number of striking features exhibited by the solutions. It should first be noted that the behavior at low Prandtl numbers and low Reynolds numbers, or whenever the diffusivity function E is small, is asymptotically that of the laminar functions. The laminar functions S_n decrease with n like $1/n$, but the turbulent functions do not decrease as rapidly. In fact, for $Pr = 30$ there is very little difference between S_1 and S_5. The higher harmonics are therefore of more importance in turbulent flow than in laminar flow, particularly at high Prandtl and Reynolds numbers, i.e. whenever E is large over most of the flow.

Although the circumferential functions for low Prandtl number turbulent flow are essentially the laminar functions, the *mean* functions S_o differ. This is due to the fact that the circumferential functions are *independent* of the *velocity* profile, while the Nusselt number for uniform flux *does* depend on velocity. It is rather surprising to observe that the ratio S_1/S_o, which in a sense is a measure of the relative importance of the first harmonic and the average, is actually greater for turbulent flow with $Pr = 0$ than in laminar flow! This means that for a given heat flux distribution the circumferential effects will be *more* pronounced in turbulent low Prandtl number flow than in laminar flow. Of equally striking importance is the observation that, except at very high Prandtl numbers, S_1 is larger than S_o, which means that the effects in a tube with marked circumferential variation in the heat flux will be quite important.

EXAMPLE

As an illustration let us consider gas flow in a reactor tube at $Re = 100\,000$, and suppose that the circumferential heat flux distribution is

$$q''(\theta) = q_o''(1 + 0{\cdot}5 \cos \theta). \quad (23)$$

Using (17), we find that the local temperature difference is given by

$$\frac{t_w - t_m}{q_o'' r_o/k} = S_o + 0{\cdot}5\, S_1 \cos \theta \quad (24a)$$

$$= 0{\cdot}0112\,(1 + 0{\cdot}80 \cos \theta). \quad (24b)$$

Note that while there is a 50 per cent increase in heat flux there is an 80 per cent increase in temperature difference. This could pose serious problems if not properly considered. To emphasize this, suppose we had simply used the local heat flux, together with the uniform flux Nusselt number, to estimate the peripheral temperature variation. We would have obtained

$$\frac{t_w - t_m}{q'' r_o/k} = 0{\cdot}0112\,(1 + 0{\cdot}5 \cos \theta). \quad (24c)$$

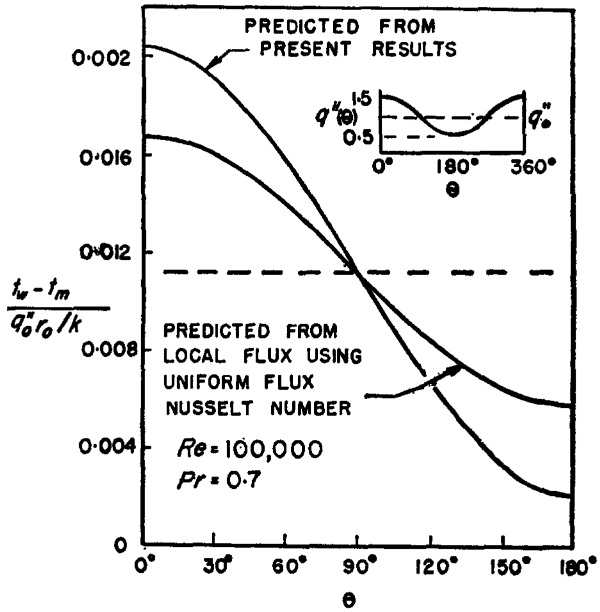

FIG. 5. A typical example.

These two predictions are compared in Fig. 5. The importance of considering the influence of the flux distribution on the heat transfer coefficient, as reflected in the functions S_n, should be evident.

CONCLUDING REMARKS

The present analysis predicts a rather surprising and important effect of circumferential heat flux variation on heat transfer in turbulent flow, particularly at low Prandtl numbers. However, the *average* temperature difference, computed from the *average* heat flux, is identical with that predicted for no peripheral flux variation. The analysis is based on the idealization that the eddy diffusivities for heat in the radial and circumferential directions are identical. Some experimental evaluation of this hypothesis would seem appropriate.

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Mr. Ward Wilkinson performed some preliminary calculations using a uniform mesh for the integration process. These proved valuable in checking the final calculations. His assistance throughout the course of this effort were appreciated.

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Résumé—Cet article présente une étude des échanges thermiques, en régimes thermique et hydrodynamique permanents, dans un tube circulaire avec flux de chaleur périphérique variable. Les résultats permettent d'évaluer les variations de température quand on chauffe le tube uniformément suivant l'axe et de façon variable suivant son périmètre. Une conclusion surprenante a été notée: l'effet de la variation du flux de chaleur pariétal est quelquefois moins important en écoulement turbulent qu'en écoulement laminaire. Un exemple montre l'importance frappante de ces effets.

Zusammenfassung—Der Wärmeübergang in hydrodynamisch und thermisch ausgebildeter Strömung in einem Kreisrohr mit veränderlichem Wärmefluss am Umfang wird analysiert. Die Ergebnisse gestatten die Bestimmung von Temperaturänderungen bei gleichmässiger Beheizung in Achsialrichtung und ungleichmässiger Beheizung über den Umfang. Als erstaunliche Schlussfolgerung ergibt sich, dass eine Änderung des Wärmestromes über den Umfang bei turbulenter Strömung manchmal grösseren Einfluss zeigt als bei Laminarströmung. Ein Beispiel verdeutlicht die Wirkung dieser Einflüsse.

Аннотация—Дается анализ гидродинамически и термодинамически полностью развитого теплообмена в круглой трубе с переменным по периметру тепловым потоком. Результаты позволяют рассчитать изменения температуры при равномерном нагреве по оси и неравномерном нагреве по периметру. Сделан неожиданный вывод о том, что влияние изменений по периметру теплового потока при турубулентном течении становится иногда более значительным, чем при ламинарном. Приводится пример, иллюстрирующий большое значение этого влияния.